Mīmāṃsā deontic logic: proof theory and applications

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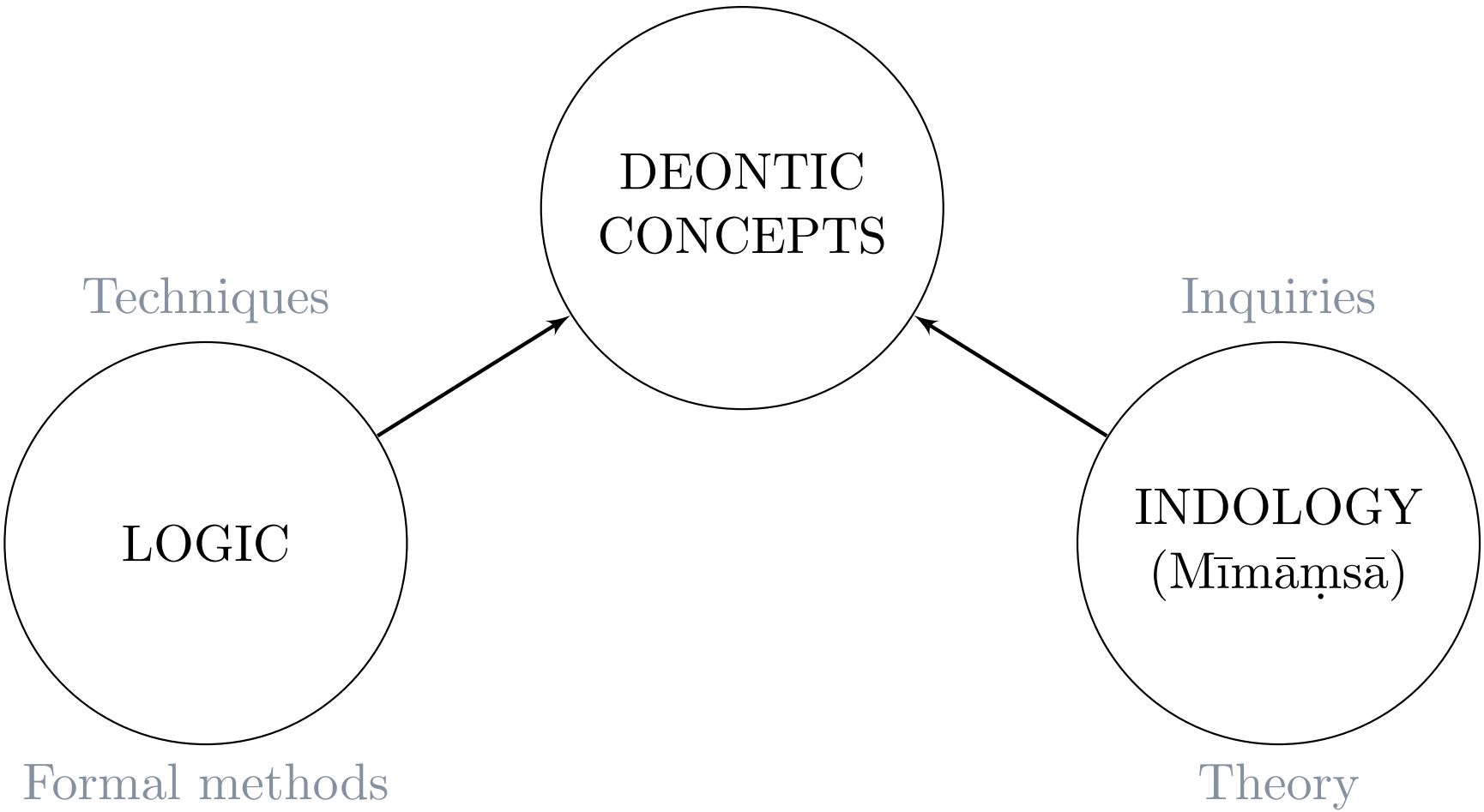
logics Logical Methods in Computer Science

The Big Picture



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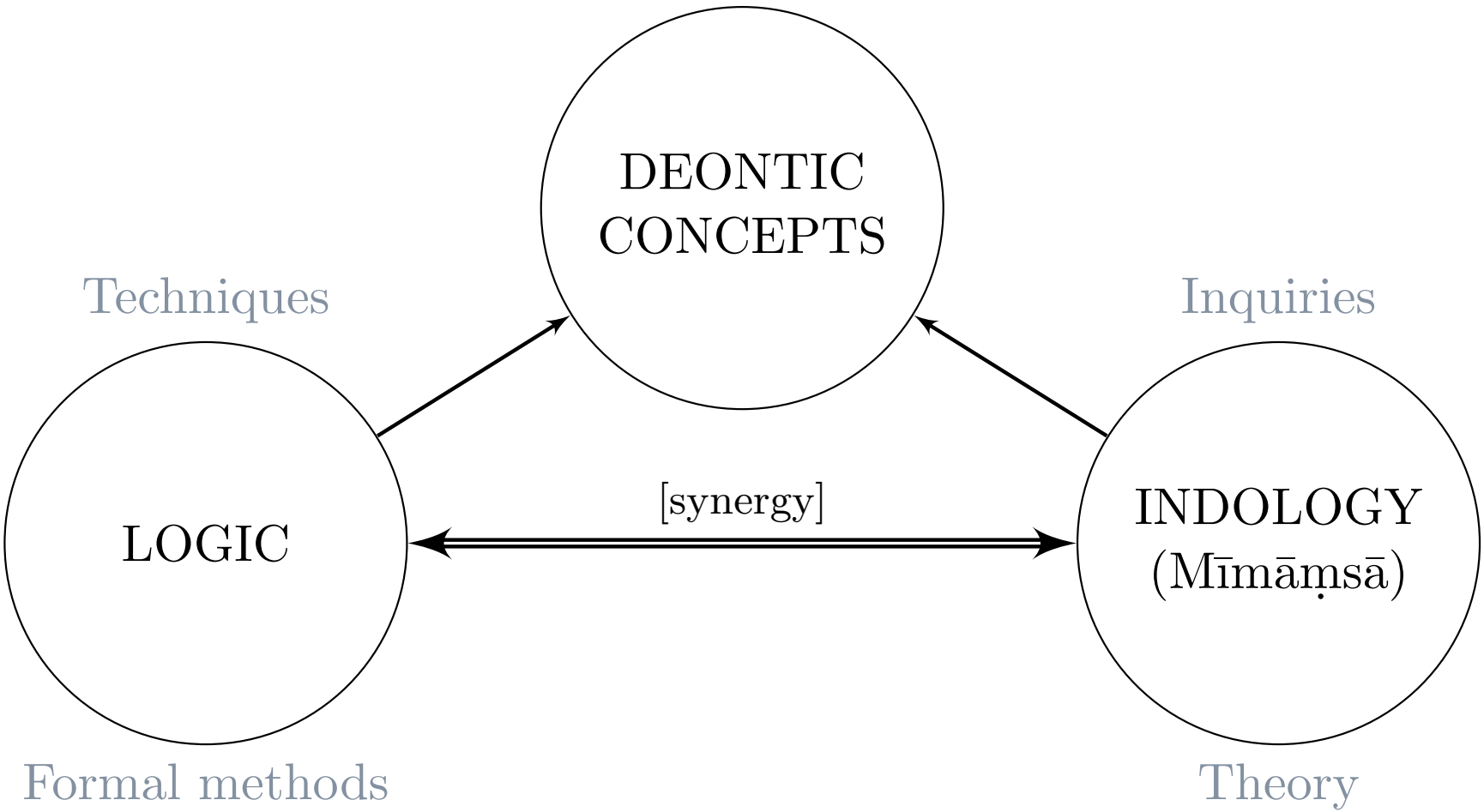
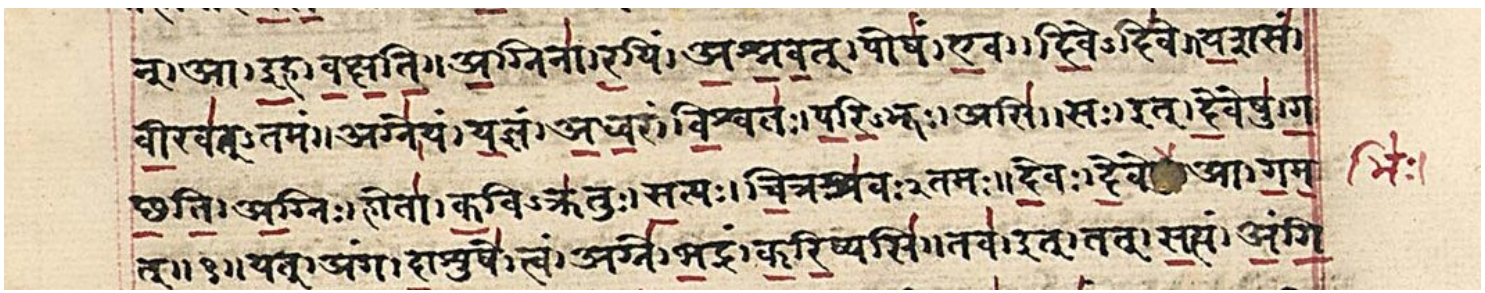


The Mīmāṃsā School

Mīmāṃsā (last centuries BCE - beginning of 20^{th} c.)

Indian school of philosophy focused on the **interpretation of the Vedas** (sacred texts, II - I millennium BCE).



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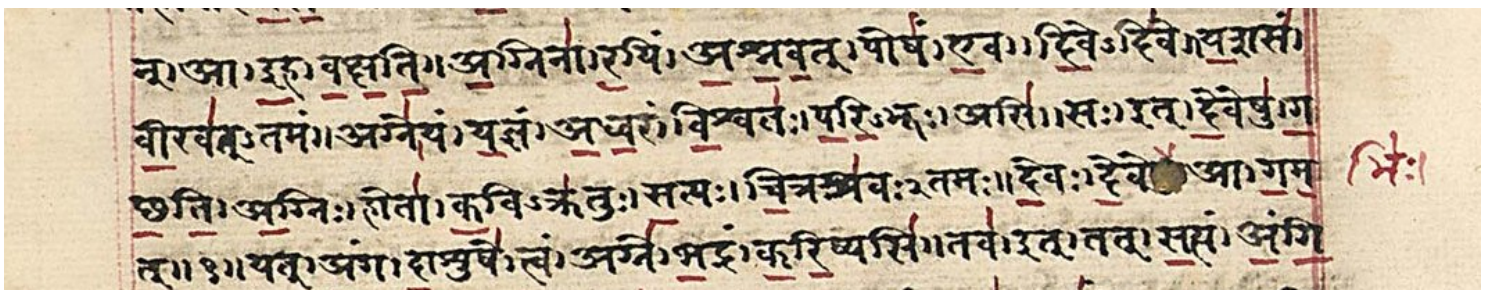

Analysis of the **prescriptive** portions.

Principles and Texts

Interpretative principles (*nyāyas*)

Rules formulated for the **interpretation** of Vedic **prescriptions**: hermeneutic, linguistic and **deontic**.

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Main sources:

- **Pūrva Mīmāṃsā Sūtra** (**PMS**) by Jaimini, last centuries BCE;
- **Śābarabhāṣya** (**ŚBh**) by Śabara, first centuries CE.

Why Logic?

Inferential reasoning

It was **employed** and **discussed** by Mīmāṃsā authors: concept of **chain of inferences**.

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Inferential reasoning

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A central concern: the **absence of contradictions**.

1.

श्येनेनाभिचरन् यजेत

2.

न हिंस्यात् सर्वा भूतानि

1.

If one wants to harm his enemy, one must perform the Śyena sacrifice

2.

One must not perform violence on any living being

The Śyena controversy

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If one wants to harm his enemy, one must perform the Śyena sacrifice

2.

One must not perform violence on any living being

1. and 2. **cannot be contradictory**, because the Vedas are not (by assumption).

Many different **explanations** have been proposed.

A Deontic Logic of Mīmāṃsā

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Basic Mīmāṃsā Deontic Logic (bMDL)

Formalisation of the **deontic system employed** by the Mīmāṃsā school.

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Formalisation of the **deontic system employed** by the Mīmāṃsā school.

Extraction of a new logic **from *nyāya*s**.

The Logic bMDL

Ingredients:

- **Classical Logic**
  - *Reductio ad absurdum* is admitted by Mīmāṃsā authors.

> *When there is a contradiction (φ and not φ), at the denial of one alternative, the other is known (to be true).*
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> Jayanta's Nyāyamañjarī, 9^{th} c. CE

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- **Dyadic deontic operator $\mathcal{O}(\ /\ )$**
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- **Modal operator $\Box$** (logic S4)

From *Nyāyas* to Hilbert Axioms

The properties of the **deontic operator** $\mathcal{O}(\ /\ )$ are extracted from *nyāyas*.

From *Nyāya*s to Hilbert Axioms, an Example

यत्र तूत्पत्त्यादयो न विध्यन्तरसिद्धास् तत्र स्वयमेव
स्वसम्बन्धिनामुत्पत्त्यादिचतुष्टयं करोति

Rāmānujācārya's *Tantrarahasya* IV.4.3.3 (14^{th} c. CE)

From *Nyāya*s to Hilbert Axioms, an Example

After many interactions with Indologists:

> *If a prescription enjoins something which has requirements, then it enjoins the requirements as well.*

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Axiom (1):

$$\Box(\varphi \to \psi) \wedge \mathcal{O}(\varphi/\theta) \to \mathcal{O}(\psi/\theta)$$

Basic Mīmāṃsā Deontic Logic (bMDL)

The logic **bMDL** extends any Hilbert system for **S4** with the following axioms:

Mīmāṃsā axioms

(1) $\Box(\varphi \to \psi) \wedge \mathcal{O}(\varphi/\theta) \to \mathcal{O}(\psi/\theta)$

(Rāmānujācārya's *Tantrarahasya* IV.4.3.3)

(2) $\Box(\psi \to \neg\varphi) \to \neg(\mathcal{O}(\varphi/\theta) \wedge \mathcal{O}(\psi/\theta))$

(Kumārila's *Tantravārttika* on PMS 1.3.3)

(3) $\Box((\chi \to \theta) \wedge (\theta \to \chi)) \wedge \mathcal{O}(\varphi/\chi) \to \mathcal{O}(\varphi/\theta)$

(ŚBh on PMS 6.1.25)

Essential Logical Questions

Use of bMDL for **effective reasoning**

Is bMDL **consistent**? Is it **decidable**? How **complex** is it?

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We employed the method in (Lellmann & Pattinson 2013) to define a **cut free sequent system** for bMDL:

$$\mathrm{G}_{\mathrm{bMDL}}$$

The System $\mathrm{G_{bMDL}}$

Standard propositional sequent rules.

Modal rules of $\mathrm{G_{bMDL}}$:

$$\frac{\Gamma^{\Box} \Rightarrow \varphi}{\Gamma \Rightarrow \Box\varphi, \Delta}\ 4 \qquad \frac{\Gamma, \Box\varphi, \varphi \Rightarrow \Delta}{\Gamma, \Box\varphi \Rightarrow \Delta}\ \mathrm{T}$$

$$\frac{\Gamma^{\Box}, \varphi \Rightarrow \psi \quad \Gamma^{\Box}, \theta \Rightarrow \chi \quad \Gamma^{\Box}, \chi \Rightarrow \theta}{\Gamma, \mathcal{O}(\varphi/\theta) \Rightarrow \mathcal{O}(\psi/\chi), \Delta}\ \mathrm{Mon}$$

$$\frac{\Gamma^{\Box}, \varphi \Rightarrow}{\Gamma, \mathcal{O}(\varphi/\theta) \Rightarrow \Delta}\ \mathrm{D_1} \qquad \frac{\Gamma^{\Box}, \varphi, \psi \Rightarrow \quad \Gamma^{\Box}, \theta \Rightarrow \chi \quad \Gamma^{\Box}, \chi \Rightarrow \theta}{\Gamma, \mathcal{O}(\varphi/\theta), \mathcal{O}(\psi/\chi) \Rightarrow \Delta}\ \mathrm{D_2}$$

where $\Gamma^{\Box}$ contains all formulae of the form $\Box\xi$ contained in Γ.

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Properties of our Calculus

- The system $\mathrm{G_{bMDL}} + \mathrm{Cut}$ is **sound and complete** for the logic **bMDL**.

$$\frac{\Gamma \Rightarrow \varphi, \Delta \quad \Sigma, \varphi \Rightarrow \Pi}{\Gamma, \Sigma \Rightarrow \Delta, \Pi}\ \mathrm{Cut}$$

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  - **Corollary**. The logic **bMDL** is **consistent**: $\bot \notin \mathrm{bMDL}$.
- The termination of the *proof search procedure* guarantees the **decidability** of the logic.
- $\mathrm{bMDL} \in$ **EXPTIME**.

Back to the Śyena Controversy

1. "*If one wants to harm his enemy, one must perform the Śyena sacrifice*"
2. "*One must not perform violence on any living being*"

- The Śyena harms the enemy
- The enemy is a living being

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2. "*One must not perform violence on any living being*" $\rightsquigarrow \mathcal{O}(\neg\text{harm}/\top)$
   - The Śyena harms the enemy, hence: $\text{syena} \rightarrow \text{harm\_e}$
   - The enemy is a living being, hence: $\text{harm\_e} \rightarrow \text{harm}$

A Syntactical Viewpoint on the Śyena Controversy

Proposition. *From the following set, in bMDL,* $\perp$ *cannot be derived:*

$\{\text{harm\_e} \to \text{harm},\ \text{syena} \to \text{harm\_e},\ \mathcal{O}(\neg\text{harm}/\top),\ \mathcal{O}(\text{syena}/\text{des\_harm})\}$

A Semantics

Countermodels, insights and explanations.

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Semantics for **bMDL**:

- S4 **frame semantics** for $\Box$ (transitive and reflexive accessibility relation);
- **neighbourhood semantics** for $\mathcal{O}$ (each neighbourhood contains pairs of sets of worlds, only accessible worlds are considered).

An *Adhikāra*-based Model

We defined a **model** for

$$\Box(\text{harm\_e} \to \text{harm}) \wedge \Box(\text{syena} \to \text{harm\_e}) \wedge \Box(\mathcal{O}(\neg\text{harm}/\top)) \wedge \Box(\mathcal{O}(\text{syena}/\text{des\_harm}))$$

The existence of this model proves that $\bot$ cannot be derived in bMDL from the set
$\{\text{harm\_e} \to \text{harm},\ \text{syena} \to \text{harm\_e},\ \mathcal{O}(\neg\text{harm}/\top),\ \mathcal{O}(\text{syena}/\text{des\_harm})\}$

$$\left(\begin{array}{l} \textbf{Weak Deduction Theorem.}\ \textit{For every sequent}\ \Gamma \Rightarrow \Delta\textit{:} \\ \{\Rightarrow \varphi \mid \varphi \in \mathcal{A}\} \vdash_{\text{GDLCut}} \Gamma \Rightarrow \Delta \quad \textit{iff} \quad \vdash_{\text{GDL}} \Box\mathcal{A}, \Gamma \Rightarrow \Delta. \end{array}\right)$$

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The model is based on the concept of ***adhikāra*** (*desires* $\rightsquigarrow$ *responsibility* $\rightsquigarrow$ *agency*).

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A **world** of the model

- represents a **possible state** w.r.t. *adhikāra*;
- is a possible combination of relevant elements: desires, outcomes of prescriptions, and actions.

E.g., $\{\text{harm}, \text{harm\_e}, \text{des\_harm}\}$, $\{\text{harm}, \text{syena}, \text{des\_harm}\}$.

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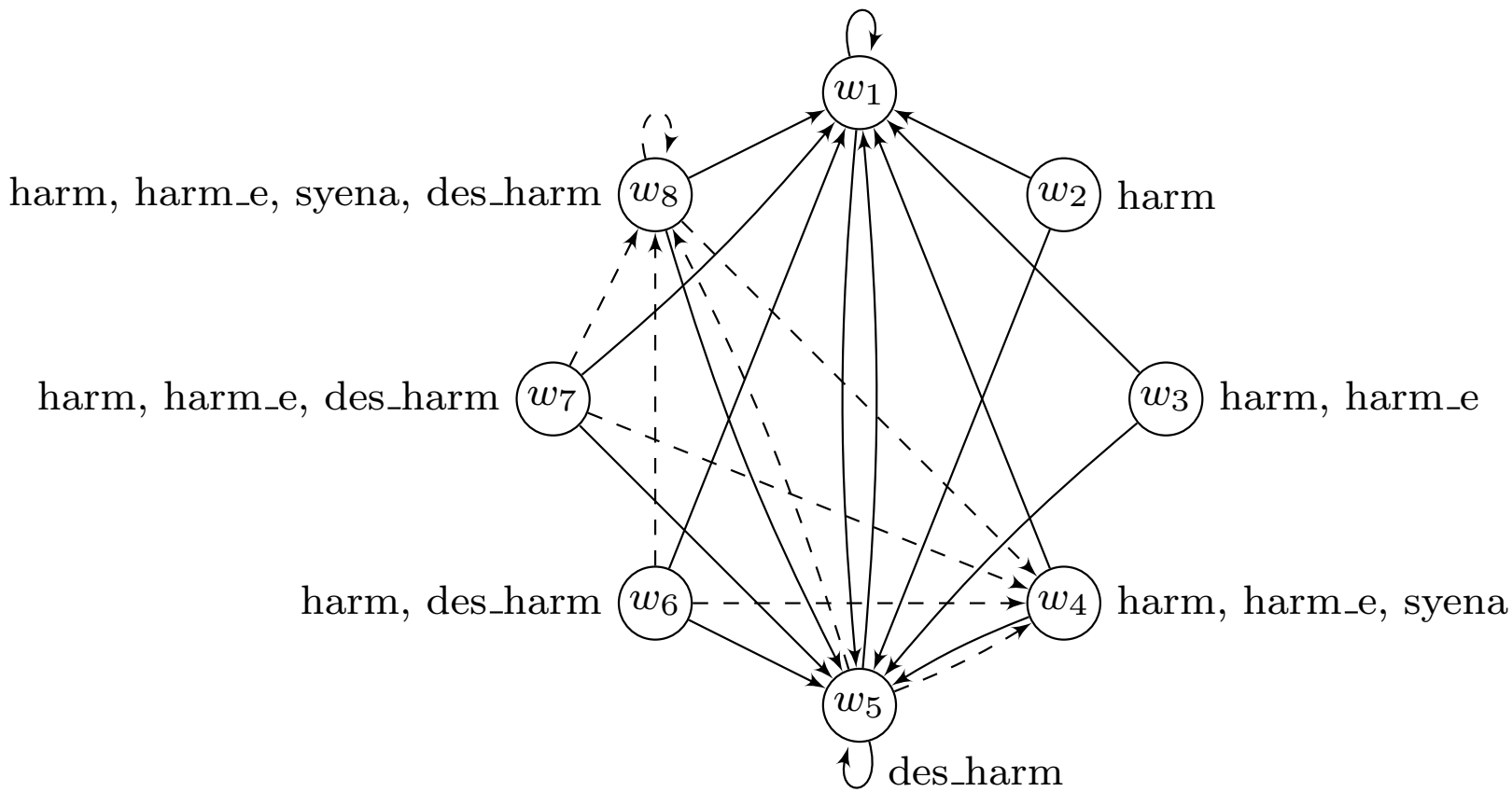

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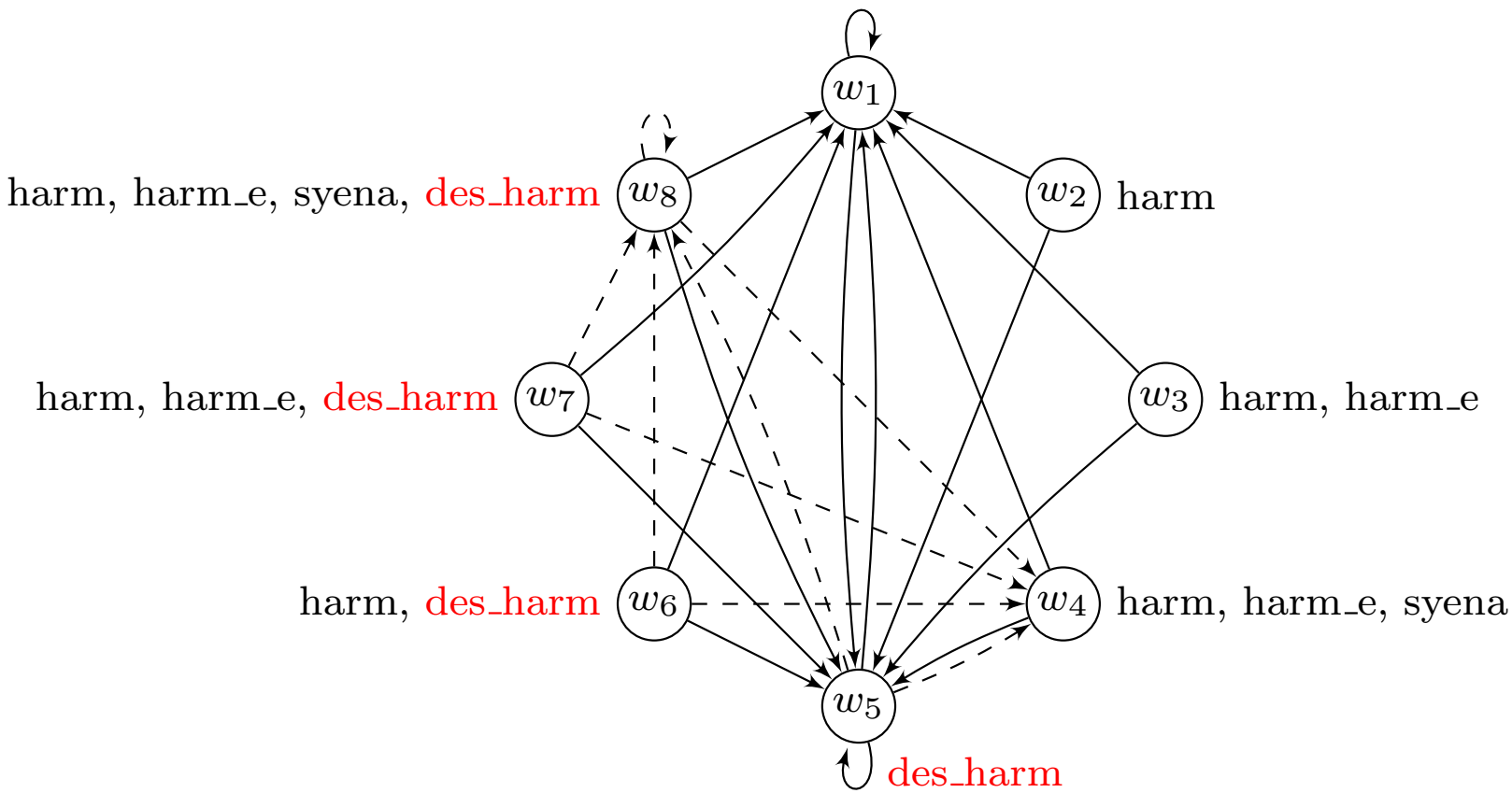

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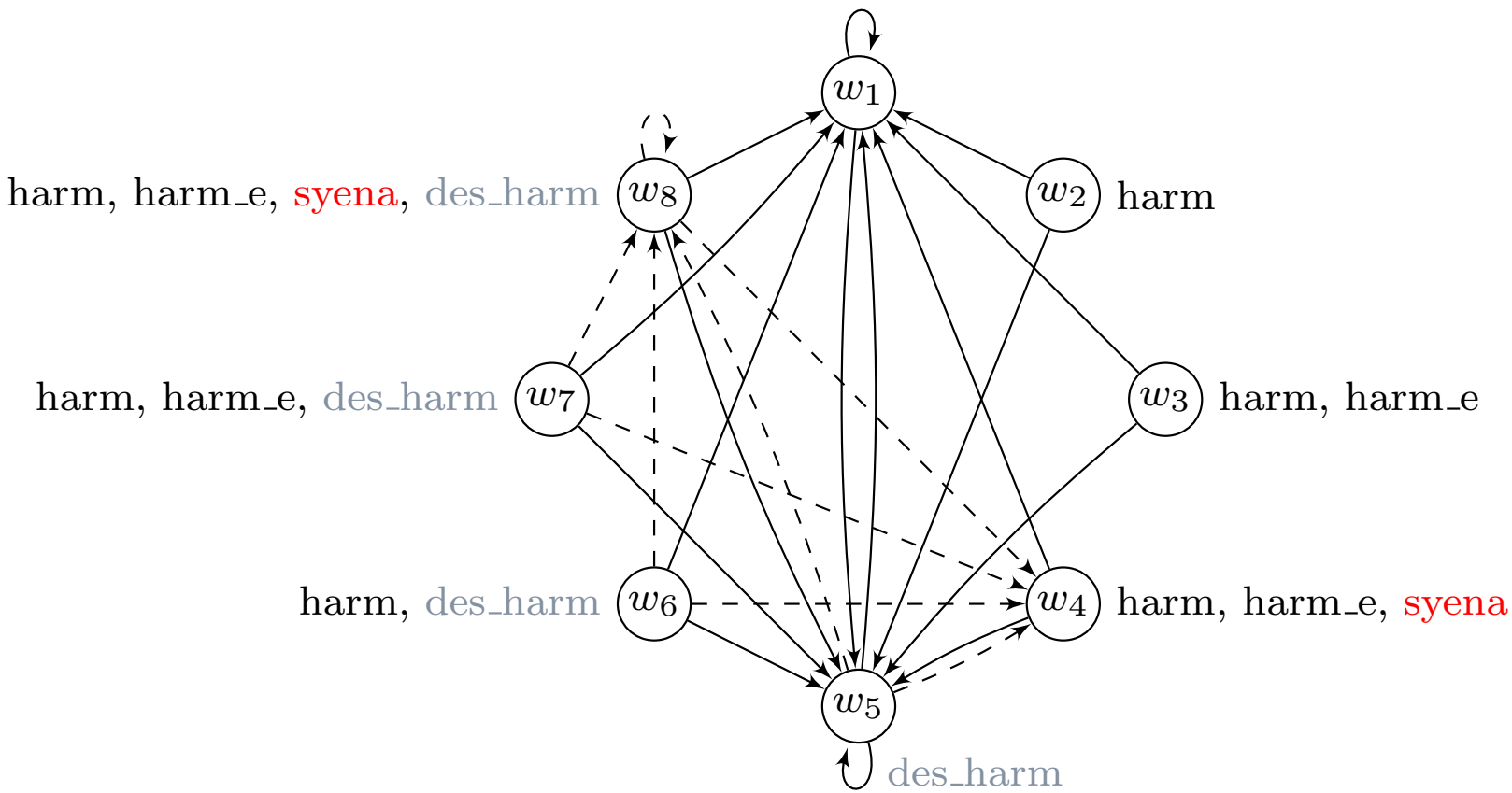

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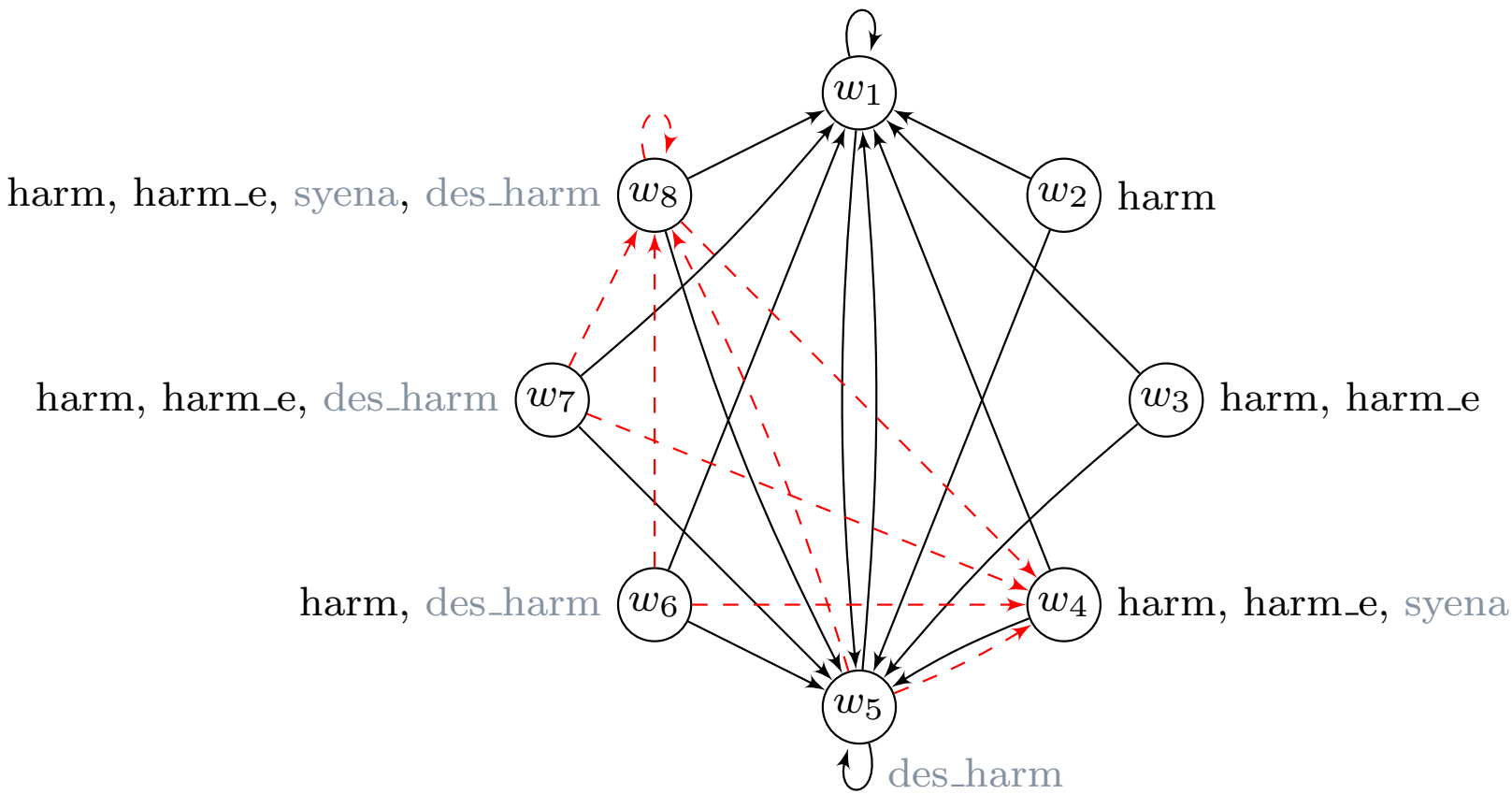
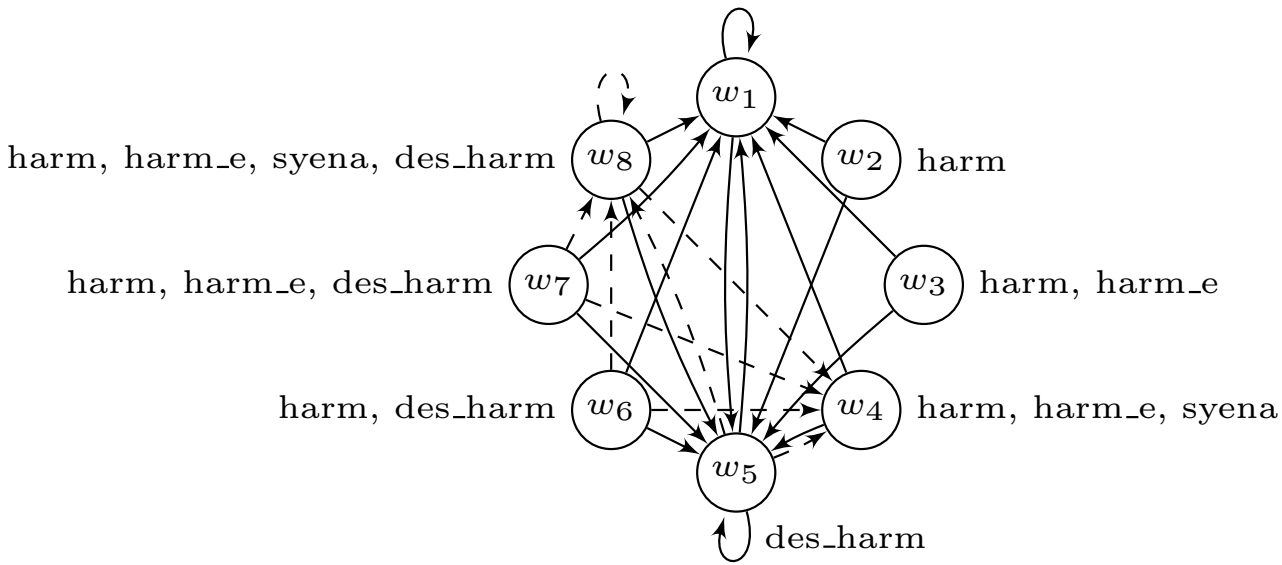


The Indological Reading



The Vedic state

Staying in w_1 all prescriptions are fulfilled and no conflict occurs.

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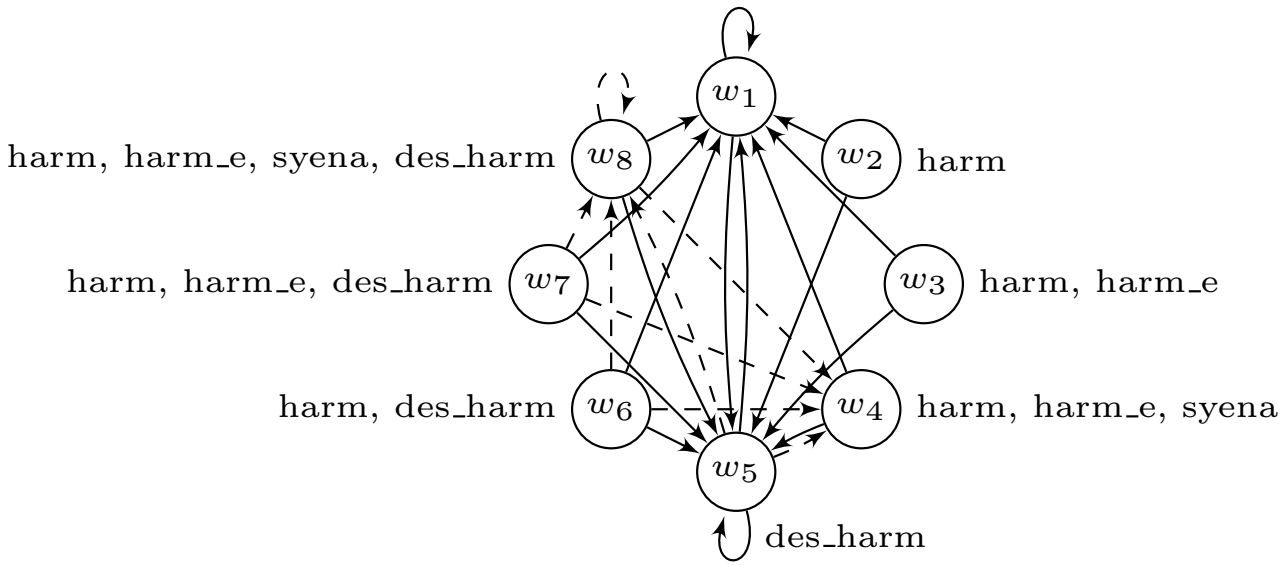

Prabhākara's solution (7^{th} c. CE)

"*A prescription regards what has to be done. But it does not say that it has to be done*" (Bṛhatī I, p. 38, l. 8f).

Future Work

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Possible **extensions** of bMDL:

- **First-order quantification**
  The agent of a duty needs to be the one identified by a given prescription (PMS 6.1.1–3).
- **Temporal operators**
  Distinction between different repetitions of the same action.
- **Handling of different authorities**
  The Vedas prevail over other authoritative texts (ŚBh 1.1.1).
- Distinction between **Obligations and Prohibitions**

Different logics for different **authors**