Comparing Strong and Weak Łukasiewicz Logic Connectives

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The logic contains the connectives
$$\wedge, \rightarrow, \neg, \vee, \otimes, \oplus$$
and truth values
$$\mathbf{0}, {}^{\mathbf{1}}\!/\mathbf{2}, \mathbf{1}.$$

The truth value $\mathbf{1}$ is designated.

Proposition 1 *The formula $((A \wedge B) \rightarrow A)$ is a tautology.*

Proposition 2 *The formula $((A \otimes B) \rightarrow A)$ is a tautology.*

Proposition 3 *The formula $((A \wedge B) \rightarrow B)$ is a tautology.*

Proposition 4 *The formula $((A \otimes B) \rightarrow B)$ is a tautology.*

Proposition 5 *The formula $((A \rightarrow B) \rightarrow ((A \rightarrow C) \rightarrow (A \rightarrow (B \wedge C))))$ is a tautology.*

Proposition 6 *The formula $((A \rightarrow B) \rightarrow ((A \rightarrow C) \rightarrow (A \rightarrow (B \otimes C))))$ is **not** a tautology.*

Proposition 7 *The formula $(A \rightarrow (A \vee B))$ is a tautology.*

Proposition 8 *The formula $(A \rightarrow (A \oplus B))$ is a tautology.*

Proposition 9 *The formula $(B \rightarrow (A \vee B))$ is a tautology.*

Proposition 10 *The formula $(B \rightarrow (A \oplus B))$ is a tautology.*

Proposition 11 *The formula $((B \rightarrow A) \rightarrow ((C \rightarrow A) \rightarrow ((B \vee C) \rightarrow A)))$ is a tautology.*

Proposition 12 *The formula $((B \rightarrow A) \rightarrow ((C \rightarrow A) \rightarrow ((B \oplus C) \rightarrow A)))$ is **not** a tautology.*

Proposition 13 *The formula $(A \vee \neg A)$ is **not** a tautology.*

Proposition 14 *The formula $(A \oplus \neg A)$ is a tautology.*

Proposition 15 *The formula $(\neg A \vee \neg\neg A)$ is **not** a tautology.*

Proposition 16 *The formula $(\neg A \oplus \neg\neg A)$ is a tautology.*

Proposition 17 *The formula $((A \rightarrow B) \vee (B \rightarrow A))$ is a tautology.*

Proposition 18 *The formula $((A \rightarrow B) \oplus (B \rightarrow A))$ is a tautology.*

Proposition 19 *The formula $((\neg A \rightarrow (B \vee C)) \rightarrow ((\neg A \rightarrow B) \vee (\neg A \rightarrow C)))$ is a tautology.*

Proposition 20 *The formula $((\neg A \to (B \oplus C)) \to ((\neg A \to B) \oplus (\neg A \to C)))$ is a tautology.*

Proposition 21 *The formula $((A \land (A \to B)) \to B)$ is **not** a tautology.*

Proposition 22 *The formula $((A \otimes (A \to B)) \to B)$ is a tautology.*

Proposition 23 *The following consequence holds:*

$$(A \lor B), \neg A \vdash B$$

Proposition 24 *The following consequence holds:*

$$(A \oplus B), \neg A \vdash B$$

Proposition 25 *The following consequence holds:*

$$(\neg C \lor \neg D), (A \to C), (B \to D) \vdash (\neg A \lor \neg B)$$

Proposition 26 *The following consequence holds:*

$$(\neg C \oplus \neg D), (A \to C), (B \to D) \vdash (\neg A \oplus \neg B)$$

Proposition 27 *The following consequence holds:*

$$(A \lor B), (A \to C), (B \to D) \vdash (C \lor D)$$

Proposition 28 *The following consequence holds:*

$$(A \oplus B), (A \to C), (B \to D) \vdash (C \oplus D)$$

Proposition 29 *The following consequence **does not** hold:*

$$(A \to (B \to C)) \vdash ((A \land B) \to C)$$

Proposition 30 *The following consequence holds:*

$$(A \to (B \to C)) \vdash ((A \otimes B) \to C)$$

Proposition 31 *The following consequence holds:*

$$((A \land B) \to C) \vdash (A \to (B \to C))$$

Proposition 32 *The following consequence holds:*

$$((A \otimes B) \to C) \vdash (A \to (B \to C))$$

Proposition 33 *The following consequence holds:*

$$(A \to B), (A \to C) \vdash (A \to (B \land C))$$

Proposition 34 *The following consequence **does not** hold:*

$$(A \to B), (A \to C) \vdash (A \to (B \otimes C))$$

Proposition 35 *The following consequence holds:*

$$((A \lor B) \to C) \vdash ((A \to C) \land (B \to C))$$

Proposition 36 *The following consequence holds:*

$$((A \vee B) \to C) \vdash ((A \to C) \otimes (B \to C))$$

Proposition 37 *The following consequence holds:*

$$((A \oplus B) \to C) \vdash ((A \to C) \wedge (B \to C))$$

Proposition 38 *The following consequence holds:*

$$((A \oplus B) \to C) \vdash ((A \to C) \otimes (B \to C))$$

Proposition 39 *The formulas $(A \wedge (B \vee C))$ and $((A \wedge B) \vee (A \wedge C))$ are equivalent.*

Proposition 40 *The formulas $(A \wedge (B \oplus C))$ and $((A \wedge B) \oplus (A \wedge C))$ are **not** equivalent.*

Proposition 41 *The formulas $(A \otimes (B \vee C))$ and $((A \otimes B) \vee (A \otimes C))$ are equivalent.*

Proposition 42 *The formulas $(A \otimes (B \oplus C))$ and $((A \otimes B) \oplus (A \otimes C))$ are **not** equivalent.*

Proposition 43 *The formulas $((B \vee C) \wedge A)$ and $((B \wedge A) \vee (C \wedge A))$ are equivalent.*

Proposition 44 *The formulas $((B \oplus C) \wedge A)$ and $((B \wedge A) \oplus (C \wedge A))$ are **not** equivalent.*

Proposition 45 *The formulas $((B \vee C) \otimes A)$ and $((B \otimes A) \vee (C \otimes A))$ are equivalent.*

Proposition 46 *The formulas $((B \oplus C) \otimes A)$ and $((B \otimes A) \oplus (C \otimes A))$ are **not** equivalent.*

Proposition 47 *The following consequence holds:*

$$(A \wedge B) \vdash A$$

Proposition 48 *The following consequence holds:*

$$(A \otimes B) \vdash A$$

Proposition 49 *The following consequence holds:*

$$(A \wedge B) \vdash B$$

Proposition 50 *The following consequence holds:*

$$(A \otimes B) \vdash B$$

Proposition 51 *The following consequence holds:*

$$A, B \vdash (A \wedge B)$$

Proposition 52 *The following consequence holds:*

$$A, B \vdash (A \otimes B)$$

Proposition 53 *The following consequence holds:*

$$A \vdash (A \vee B)$$

Proposition 54 *The following consequence holds:*

$$A \vdash (A \oplus B)$$

Proposition 55 *The following consequence holds:*

$$(A \vee B) \vdash (B \vee A)$$

Proposition 56 *The following consequence holds:*

$$(A \oplus B) \vdash (B \oplus A)$$

Proposition 57 *The following consequence holds:*

$$(A \vee A) \vdash A$$

Proposition 58 *The following consequence **does not** hold:*

$$(A \oplus A) \vdash A$$

Proposition 59 *The following consequence holds:*

$$(A \vee (B \vee C)) \vdash ((A \vee B) \vee C)$$

Proposition 60 *The following consequence holds:*

$$(A \oplus (B \oplus C)) \vdash ((A \oplus B) \oplus C)$$

Proposition 61 *The following consequence holds:*

$$(A \vee (B \wedge C)) \vdash ((A \vee B) \wedge (A \vee C))$$

Proposition 62 *The following consequence holds:*

$$(A \oplus (B \wedge C)) \vdash ((A \oplus B) \wedge (A \oplus C))$$

Proposition 63 *The following consequence holds:*

$$(A \vee (B \otimes C)) \vdash ((A \vee B) \otimes (A \vee C))$$

Proposition 64 *The following consequence holds:*

$$(A \oplus (B \otimes C)) \vdash ((A \oplus B) \otimes (A \oplus C))$$

Proposition 65 *The following consequence holds:*

$$((A \vee B) \wedge (A \vee C)) \vdash (A \vee (B \wedge C))$$

Proposition 66 *The following consequence holds:*

$$((A \oplus B) \wedge (A \oplus C)) \vdash (A \oplus (B \wedge C))$$

Proposition 67 *The following consequence holds:*

$$((A \vee B) \otimes (A \vee C)) \vdash (A \vee (B \otimes C))$$

Proposition 68 *The following consequence **does not** hold:*

$$((A \oplus B) \otimes (A \oplus C)) \vdash (A \oplus (B \otimes C))$$

Proposition 69 *The following consequence holds:*

$$(A \vee C) \vdash (\neg\neg A \vee C)$$

Proposition 70 *The following consequence holds:*

$$(A \oplus C) \vdash (\neg\neg A \oplus C)$$

Proposition 71 *The following consequence holds:*

$$(\neg\neg A \vee C) \vdash (A \vee C)$$

Proposition 72 *The following consequence holds:*

$$(\neg\neg A \oplus C) \vdash (A \oplus C)$$

Proposition 73 *The following consequence holds:*

$$(\neg(A \vee B) \vee C) \vdash ((\neg A \wedge \neg B) \vee C)$$

Proposition 74 *The following consequence holds:*

$$(\neg(A \oplus B) \oplus C) \vdash ((\neg A \wedge \neg B) \oplus C)$$

Proposition 75 *The following consequence holds:*

$$(\neg(A \vee B) \vee C) \vdash ((\neg A \otimes \neg B) \vee C)$$

Proposition 76 *The following consequence holds:*

$$(\neg(A \oplus B) \oplus C) \vdash ((\neg A \otimes \neg B) \oplus C)$$

Proposition 77 *The following consequence holds:*

$$((\neg A \wedge \neg B) \vee C) \vdash (\neg(A \vee B) \vee C)$$

Proposition 78 *The following consequence **does not** hold:*

$$((\neg A \wedge \neg B) \oplus C) \vdash (\neg(A \oplus B) \oplus C)$$

Proposition 79 *The following consequence holds:*

$$((\neg A \otimes \neg B) \vee C) \vdash (\neg(A \vee B) \vee C)$$

Proposition 80 *The following consequence holds:*

$$((\neg A \otimes \neg B) \oplus C) \vdash (\neg(A \oplus B) \oplus C)$$

Proposition 81 *The following consequence holds:*

$$(\neg(A \wedge B) \vee C) \vdash ((\neg A \vee \neg B) \vee C)$$

Proposition 82 *The following consequence holds:*

$$(\neg(A \wedge B) \oplus C) \vdash ((\neg A \oplus \neg B) \oplus C)$$

Proposition 83 *The following consequence **does not** hold:*

$$(\neg(A \otimes B) \vee C) \vdash ((\neg A \vee \neg B) \vee C)$$

Proposition 84 *The following consequence holds:*

$$(\neg(A \otimes B) \oplus C) \vdash ((\neg A \oplus \neg B) \oplus C)$$

Proposition 85 *The following consequence holds:*

$$((\neg A \vee \neg B) \vee C) \vdash (\neg(A \wedge B) \vee C)$$

Proposition 86 *The following consequence **does not** hold:*

$$((\neg A \oplus \neg B) \oplus C) \vdash (\neg(A \wedge B) \oplus C)$$

Proposition 87 *The following consequence holds:*

$$((\neg A \vee \neg B) \vee C) \vdash (\neg(A \otimes B) \vee C)$$

Proposition 88 *The following consequence holds:*

$$((\neg A \oplus \neg B) \oplus C) \vdash (\neg(A \otimes B) \oplus C)$$

Proposition 89 *The equality $A = (A \wedge A)$ holds.*

Proposition 90 *The equality $A = (A \otimes A)$ does **not** hold.*

Proposition 91 *The equality $(A \wedge (B \vee C)) = ((A \wedge B) \vee (A \wedge C))$ holds.*

Proposition 92 *The equality $((B \vee C) \wedge A) = ((B \wedge A) \vee (C \wedge A))$ holds.*

Proposition 93 *The equality $(A \wedge (B \oplus C)) = ((A \wedge B) \oplus (A \wedge C))$ does **not** hold.*

Proposition 94 *The equality $(A \otimes (B \vee C)) = ((A \otimes B) \vee (A \otimes C))$ holds.*

Proposition 95 *The equality $((B \vee C) \otimes A) = ((B \otimes A) \vee (C \otimes A))$ holds.*

Proposition 96 *The equality $(A \otimes (B \oplus C)) = ((A \otimes B) \oplus (A \otimes C))$ does **not** hold.*

Proposition 97 *The equality $(A \rightarrow B) = (\neg A \vee B)$ does **not** hold.*

Proposition 98 *The equality $(A \rightarrow B) = (\neg A \oplus B)$ holds.*

Proposition 99 *The equality $(A \rightarrow B) = \neg(A \wedge \neg B)$ does **not** hold.*

Proposition 100 *The equality $(A \rightarrow B) = \neg(A \otimes \neg B)$ holds.*

Proposition 101 *The equality $(A \vee B) = ((A \rightarrow B) \rightarrow B)$ holds.*

Proposition 102 *The equality $(A \oplus B) = ((A \rightarrow B) \rightarrow B)$ does **not** hold.*

Proposition 103 *The equality $(A \vee B) = \neg(\neg A \wedge \neg B)$ holds.*

Proposition 104 *The equality $(A \oplus B) = \neg(\neg A \wedge \neg B)$ does **not** hold.*

Proposition 105 *The equality $(A \vee B) = \neg(\neg A \otimes \neg B)$ does **not** hold.*

Proposition 106 *The equality $(A \oplus B) = \neg(\neg A \otimes \neg B)$ holds.*

Proposition 107 *The equality $(A \wedge B) = \neg(A \rightarrow \neg B)$ does **not** hold.*

Proposition 108 *The equality $(A \otimes B) = \neg(A \rightarrow \neg B)$ holds.*

Proposition 109 *The equality $(A \wedge B) = \neg(\neg A \vee \neg B)$ holds.*

Proposition 110 *The equality $(A \wedge B) = \neg(\neg A \oplus \neg B)$ does **not** hold.*

Proposition 111 *The equality $(A \otimes B) = \neg(\neg A \vee \neg B)$ does **not** hold.*

Proposition 112 *The equality $(A \otimes B) = \neg(\neg A \oplus \neg B)$ holds.*

Proposition 113 *The equality $\neg(A \vee B) = (\neg A \wedge \neg B)$ holds.*

Proposition 114 *The equality $\neg(A \oplus B) = (\neg A \wedge \neg B)$ does **not** hold.*

Proposition 115 *The equality $\neg(A \vee B) = (\neg A \otimes \neg B)$ does **not** hold.*

Proposition 116 *The equality $\neg(A \oplus B) = (\neg A \otimes \neg B)$ holds.*

Proposition 117 *The equality $\neg(A \wedge B) = (\neg A \vee \neg B)$ holds.*

Proposition 118 *The equality $\neg(A \wedge B) = (\neg A \oplus \neg B)$ does **not** hold.*

Proposition 119 *The equality $\neg(A \otimes B) = (\neg A \vee \neg B)$ does **not** hold.*

Proposition 120 *The equality $\neg(A \otimes B) = (\neg A \oplus \neg B)$ holds.*

Proposition 121 *The following meta-consequence **does not** hold:*

$$(P \wedge Q) \vdash R \quad / \quad P \vdash (Q \to R)$$

Proposition 122 *The following meta-consequence **does not** hold:*

$$(P \otimes Q) \vdash R \quad / \quad P \vdash (Q \to R)$$

1 Program listing: `ex_lukasiewicz2.pl`

```
% Test file to compare strong and weak Lukasiewicz operators

% make sure MUltseq is loaded
:- ensure_loaded('../multseq/multseq').

% load sample properties
:- [properties].

% load the rules
:- load_logic('lukasiewicz.msq').

% define standard Omap
:- setOmap([(neg)/(-),imp/(>),and/(/\),or/(\/),equiv/(=)]).

% check all properties and write report to out.tex

:- set_option(tex_output(terse)).

:- start_logging(ex_lukasiewicz2,'.tex').

:- print_tex(tex_title("Comparing Strong and Weak \\L ukasiewicz Logic Connectives")).

:- print_tex(tex_logic).

:- (compareProp([[and,sand]/(/\),[or,sor]/(\/)], _), fail); true.

:- print_tex(tex_listing("ex_lukasiewicz2.pl")).

:- stop_logging.
```