LNSprover*

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LNSprover implements backwards proof search in modular linear nested sequent calculi for normal and non-normal propositional modal logics. It is based on the article [1].

0.1 Implemented logics

- The logics in the non-normal modal cube (see Fig. 1), i.e., extensions of classical modal logic E with any combination of the axioms
  - C: regularity ($\Box A \wedge \Box B \to \Box(A \wedge B)$)
  - N: necessitation ($\Box \top$)
  - M: monotonicity ($\Box(A \wedge B) \to \Box A \wedge \Box B$)
- The logics in the modal tesseract (see Fig. 2), i.e., extensions of monotone modal logic M with any combination of the axioms
  - P: ($\neg \Box \bot$)
  - D: ($\Box \neg A \to \neg \Box A$)
  - T: ($\Box A \to A$)
  - $\mathsf{4}$: ($\Box A \to \Box \Box A$)
  - C: regularity ($\Box A \wedge \Box B \to \Box(A \wedge B)$)
  - N: necessitation ($\Box \top$)

  as well as the logics $\mathsf{M5}, \mathsf{MP5}, \mathsf{M45}, \mathsf{MP45}, \mathsf{MD45}, \mathsf{MC45}, \mathsf{MCD45}$ including the axiom

  - $\mathsf{5}$: ($\Box \neg A \to \Box \neg \Box A$)

  (see Fig. 3). Note that logic MCN is normal modal logic K, so e.g. $\mathsf{MCNT4}$ is the logic $\mathsf{KT4} = \mathsf{S4}$ in standard terminology. Also note that $\mathsf{5}$ implies N, so we have e.g. $\mathsf{MC45} = \mathsf{MCN45} = \mathsf{K45}$ and $\mathsf{MCD45} = \mathsf{MCND45} = \mathsf{KD45}$.

*Version 1.0

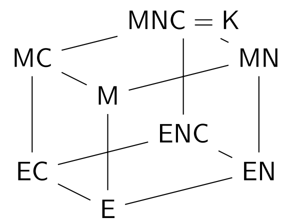

Figure 1: The non-normal modal cube

0.2 Input syntax

The syntax of formulae `F` is given by

```
P | true | false | neg F | box F | F and F | F or F | F -> F
```

where `P` is `a,b,c,...` The usual conventions for omitting parentheses are in place, i.e., the binding strength is: unary connectives > `and` > `or` > `->`.

0.3 Usage

Make sure you have SWI-Prolog installed (`http://www.swi-prolog.org/`) and the files lnsprover.pl, derivation.tex and output.tex are in the same folder. Start swipl and load the file lnsprover.pl, e.g., typing

```
swipl lnsprover.pl
```

at a terminal. Run the prover with

```
?- prv([Axioms],[Gamma],[Delta]).
```

where `Gamma` ⇒ `Delta` is the sequent you want to check and `Axioms` is a list of axioms from

```
cl, clc, cln, clm, mon, p, d, t, 4, 5, c, n
```

For extensions of classical modal logic E include `cl`, for the logics in the tesseract include `mon`. The axioms prefixed with cl are those for extensions of the non-normal cube, the remaining ones those for the tesseract. **IMPORTANT:** for logics including axiom `t` switch on Kleeneing (see Section 0.5 below).

If the sequent is derivable, the prover displays the derivation and writes it to a .tex file. If there is more than one derivation, hit ; to search for the next one. Run latex on output.tex to obtain a pdf showing all these derivations. For larger derivations you might need to increase the paper size in the preamble of output.tex by replacing the `a4paper` option in

```
\usepackage[...,a4paper]{geometry}
```

with e.g. `a0paper`.

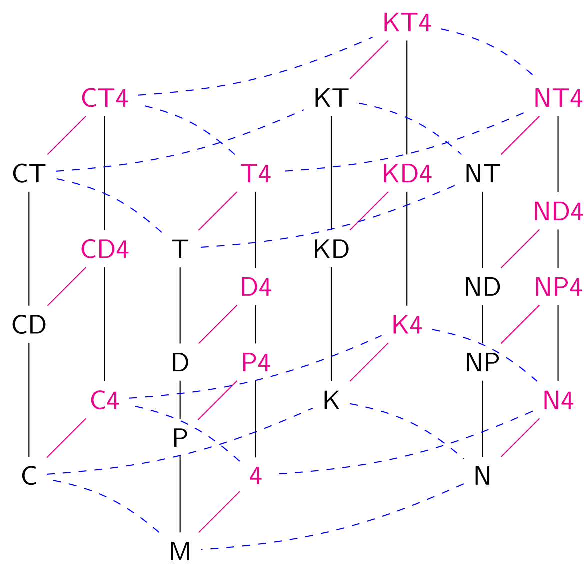

Figure 2: The modal tesseract

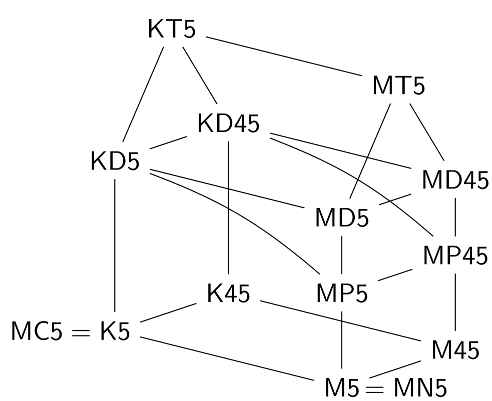

Figure 3: The extensions of modal logic M5

0.4 Example

1. To check whether the sequent

$$\Box A \wedge \Box B \Rightarrow \Box(A \wedge B)$$

is derivable in classical modal logic with axioms N and C, type

```
?- prv([cl,cln,clc],[box a and box b],[box (a and b)]).
```

2. To check whether

$$\Box(A \wedge B) \Rightarrow \Box\Box B$$

is derivable in monotone modal logic with axioms 4 and C, type

```
?- prv([mon,4,c],[box (a and b)],[box box b]).
```

0.5 Kleeneing

By default the propositional rules are not Kleene'd (i.e., the principal formulae are not copied into the premiss(es)). This can be changed by typing

```
?- kleeneing.
```

Likewise, to turn Kleeneing off again type

```
?- nokleeneing.
```

IMPORTANT: For logics including the axiom T, switch on Kleeneing to prevent loops in proof search.

References

[1] B. Lellmann and E. Pimentel. Proof search in nested sequent calculi. In M. Davis, A. Fehnker, A. McIver, and A. Voronkov, editors, *LPAR-20 2015*, pages 558–574. Springer Berlin Heidelberg, 2015.

Table 1: The propositional rules

$$\frac{}{\Gamma, A \Rightarrow A, \Delta}\ \mathsf{init} \qquad \frac{}{\Gamma, \bot \Rightarrow \Delta}\ \bot_L \qquad \frac{}{\Gamma \Rightarrow \top, \Delta}\ \top_R$$

$$\frac{\Gamma \Rightarrow A, \Delta}{\Gamma, \neg A \Rightarrow \Delta}\ \neg_L \qquad \frac{\Gamma, A \Rightarrow \Delta}{\Gamma \Rightarrow \neg A, \Delta}\ \neg_R$$

$$\frac{\Gamma, A, B \Rightarrow \Delta}{\Gamma, A \wedge B \Rightarrow \Delta}\ \wedge_L \qquad \frac{\Gamma \Rightarrow A, \Delta \quad \Gamma \Rightarrow B, \Delta}{\Gamma \Rightarrow A \wedge B, \Delta}\ \wedge_R$$

$$\frac{\Gamma, A \Rightarrow \Delta \quad \Gamma, B \Rightarrow \Delta}{\Gamma, A \vee B \Rightarrow \Delta}\ \vee_L \qquad \frac{\Gamma \Rightarrow A, B, \Delta}{\Gamma \Rightarrow A \wedge B, \Delta}\ \vee_R$$

$$\frac{\Gamma, B \Rightarrow \Delta \quad \Gamma \Rightarrow A, \Delta}{\Gamma, A \rightarrow B \Rightarrow \Delta}\ \rightarrow_L \qquad \frac{\Gamma, A \Rightarrow B, \Delta}{\Gamma \Rightarrow A \rightarrow B, \Delta}\ \rightarrow_R$$

Table 2: The contraction rules

$$\frac{\Gamma, A, A \Rightarrow \Delta}{\Gamma, A \Rightarrow \Delta}\ \mathsf{Con_L} \qquad \frac{\Gamma \Rightarrow A, A, \Delta}{\Gamma \Rightarrow A, \Delta}\ \mathsf{Con_R}$$

Table 3: The modal rules for the tesseract

$$\frac{\mathcal{G} \mathbin{/\!/} \Gamma \Rightarrow \Delta \mathbin{/\!/} \Sigma, A \Rightarrow \Pi}{\mathcal{G} \mathbin{/\!/} \Gamma, \Box A \Rightarrow \Delta \mathbin{/\!/}^{\mathsf{m}} \Sigma \Rightarrow \Pi}\ \Box^{\mathsf{m}}_L \qquad \frac{\mathcal{G} \mathbin{/\!/} \Gamma \Rightarrow \Delta \mathbin{/\!/}^{\mathsf{m}} \ \Rightarrow A}{\mathcal{G} \mathbin{/\!/} \Gamma \Rightarrow \Box A, \Delta}\ \Box^{\mathsf{m}}_R$$

$$\frac{\mathcal{G} \mathbin{/\!/}^{\mathsf{m}} \Gamma \Rightarrow \Delta}{\mathcal{G} \mathbin{/\!/} \Gamma \Rightarrow \Delta}\ \mathsf{C} \qquad \frac{\mathcal{G} \mathbin{/\!/} \Gamma \Rightarrow \Delta}{\mathcal{G} \mathbin{/\!/}^{\mathsf{m}} \Gamma \Rightarrow \Delta}\ \mathsf{N}$$

$$\frac{\mathcal{G} \mathbin{/\!/}^{\mathsf{m}} \ \Rightarrow}{\mathcal{G}}\ \mathsf{P} \qquad \frac{\mathcal{G} \mathbin{/\!/} \Gamma \Rightarrow \Delta \mathbin{/\!/}^{\mathsf{m}} A \Rightarrow}{\mathcal{G} \mathbin{/\!/} \Gamma, \Box A \Rightarrow \Delta}\ \mathsf{D} \qquad \frac{\mathcal{G} \mathbin{/\!/} \Gamma, \Box A, A \Rightarrow \Delta}{\mathcal{G} \mathbin{/\!/} \Gamma, \Box A \Rightarrow \Delta}\ \mathsf{T}$$

$$\frac{\mathcal{G} \mathbin{/\!/} \Gamma \Rightarrow \Delta \mathbin{/\!/} \Sigma, \Box A \Rightarrow \Pi}{\mathcal{G} \mathbin{/\!/} \Gamma, \Box A \Rightarrow \Delta \mathbin{/\!/}^{\mathsf{m}} \Sigma \Rightarrow \Pi}\ 4 \qquad \frac{\mathcal{G} \mathbin{/\!/} \Gamma \Rightarrow \Delta \mathbin{/\!/} \Sigma \Rightarrow \Box A, \Pi}{\mathcal{G} \mathbin{/\!/} \Gamma \Rightarrow \Box A, \Delta \mathbin{/\!/}^{\mathsf{m}} \Sigma \Rightarrow \Pi}\ 5$$

Table 4: The modal rules for the non-normal cube

$$\frac{\mathcal{G} \mathbin{/\!/} \Gamma \Rightarrow \Delta \mathbin{/\!/}^{\mathsf{efin}} [\Sigma, A \Rightarrow \Pi; \Omega \Rightarrow \Theta] \qquad \mathcal{G} \mathbin{/\!/} \Gamma \Rightarrow \Delta \mathbin{/\!/} \Omega \Rightarrow A, \Theta}{\mathcal{G} \mathbin{/\!/} \Gamma, \Box A \Rightarrow \Delta \mathbin{/\!/}^{\mathsf{e}} [\Sigma \Rightarrow \Pi; \Omega \Rightarrow \Theta]}\ \Box^{\mathsf{e}}_L$$

$$\frac{\mathcal{G} \mathbin{/\!/} \Gamma \Rightarrow \Delta \mathbin{/\!/}^{\mathsf{e}} [\ \Rightarrow A; A \Rightarrow\ ]}{\mathcal{G} \mathbin{/\!/} \Gamma \Rightarrow \Box A, \Delta}\ \Box^{\mathsf{e}}_R \qquad \frac{\mathcal{G} \mathbin{/\!/} \Gamma \Rightarrow \Delta}{\mathcal{G} \mathbin{/\!/} \Gamma \Rightarrow \Delta \mathbin{/\!/}^{\mathsf{efin}} [\Sigma \Rightarrow \Pi; \Omega \Rightarrow \Theta]}\ \mathsf{jump}$$

$$\frac{\mathcal{G} \mathbin{/\!/} \Gamma \Rightarrow \Delta \mathbin{/\!/}^{\mathsf{e}} [\Sigma \Rightarrow \Pi; \Omega, \bot \Rightarrow \Theta]}{\mathcal{G} \mathbin{/\!/} \Gamma \Rightarrow \Delta \mathbin{/\!/}^{\mathsf{e}} [\Sigma \Rightarrow \Pi; \Omega \Rightarrow \Theta]}\ \mathsf{M} \qquad \frac{\mathcal{G} \mathbin{/\!/} \Gamma \Rightarrow \Delta \mathbin{/\!/}^{\mathsf{e}} [\Sigma \Rightarrow \Pi; \Omega \Rightarrow \Theta]}{\mathcal{G} \mathbin{/\!/} \Gamma \Rightarrow \Delta \mathbin{/\!/}^{\mathsf{efin}} [\Sigma \Rightarrow \Pi; \Omega \Rightarrow \Theta]}\ \mathsf{C}$$

$$\frac{\mathcal{G} \mathbin{/\!/} \Gamma \Rightarrow \Delta \mathbin{/\!/}^{\mathsf{efin}} [\Sigma \Rightarrow \Pi; \Omega \Rightarrow \Theta]}{\mathcal{G} \mathbin{/\!/} \Gamma \Rightarrow \Delta \mathbin{/\!/}^{\mathsf{e}} [\Sigma \Rightarrow \Pi; \Omega \Rightarrow \Theta]}\ \mathsf{N}$$